Machakos University College

ISO 9001:2008 Certified

(A Constituent College of Kenyatta University)

DEPARTMENT OF MATHEMATICS AND STATISTICS

SCHOOL OF EDUCATION

SECOND YEAR FIRST SEMESTER EXAMINATION FOR THE AWARD OF BACHELOR OF EDUCATION

SMA 202: LINEAR ALGEBRA 1

Date: 2014/8/15 **Time: 8:30 – 10:30 AM**

Instructions:

Answer question ONE which is compulsory and any other TWO questions

Question ONE (30 marks)

(a) Show that

(i) $a = 2i + j - k$ and $b = -i + j - k$ are orthogonal

(ii) Evaluate $a.(b \times c)$ given that $= 2i + k$, $b = i + j + 2k$ and $c = -i + j$

(6 marks)

(b) Determine the inverse of the matrix $\begin{bmatrix} 1 & -1 & 3 \\ -1 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$ (6 marks)

(c) show that the set of vectors

$r_1 = 2a - 3b + c$,

$r_2 = 3a - 5b + 2c$,

$r_3 = 4a - 5b + c$. Given that a, b and c are non-zero and non co-plannar vectors is linearly dependent. (6 marks)

(d) If **V** a subset of R^3 consisting of all the vectors which satisfy the condition $x_1 + x_2 = 0$ and $x_2 + x_3 = 0$ show that **V** is a subspace. (6 marks)

(e) By letting A be the coefficient matrix and C an augmented matrix calculate rank (A) and rank C). State whether the systems of equations are consistent and the nature of their solutions

$3x + 4y + 8z = 15$
$x + 2y + 3z = 9$
$2x + 2y + 5z = 6$ (6 marks)

Question TWO (20 marks)

(a) Find the angles which the vector $V = 3i - 6j + 2k$ makes with the coordinate axes (5 marks)

(b) State the Cauchy-Schwartz inequality theorem (3 marks)

(c) In the brand switching challenge the transition probability matrix is given by

$$\begin{pmatrix} 0.5 & 0.4 & 0.1 \\ 0.2 & 0.7 & 0.1 \\ 0.1 & 0.2 & 0.7 \end{pmatrix}$$

If the market share vector at n^{th} stage is (0.4, 0.3, 0.3)

i. What will be the market vector in $(n+1)^{th}$ stage.

ii. Calculate the market share in the equilibrium situation. (12 marks)

Question THREE (20 marks)

(a) Given that $A = 2i + j + k$ and $B = 4i + 2j - 3k$ Find the

i. Dot product (A.B)

ii. Angle between A and B (5 marks)

(b) Show that the necessary and sufficient conditions for the vectors

$r_1 = x_1 i + y_1 j + z_1 k$,
$r_2 = x_2 i + y_2 j + z_2 k$,
$r_3 = x_3 i + y_3 j + z_3 k$ to be linearly independent is that the determinant

$$\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} \neq 0$$ (7 marks)

(c) Solve for x_1, x_2 , x_3 *and* x_4 by using Gaussian elimination method given that.

$$x_1 + x_2 + 3x_3 + x_4 = -1,$$
$$2x_1 + 2x_2 - x_3 + 2x_4 = 5,$$
$$2x_1 + 3x_2 + 4x_3 + 2x_4 = 3,$$
$$2x_1 + 3x_2 + 4x_3 + 4x_4 = 3,$$
(8 marks)

Question FOUR (20 marks)

Solve the values of x for which M is a singular matrix (8 marks)

$$\boldsymbol{M} = \begin{pmatrix} x-3 & 1 & -1 \\ -7 & x+5 & -1 \\ -6 & 6 & x-2 \end{pmatrix}$$

Q3d Determine whether $v_1 = (1,1,1)$, $v_2 = (1,-1,0)$, $v_3 = (1,\ 1,-2)$ are independent, confirm your findings on orthogonal basis. (8 marks)

Q3e Given $v_1 = \binom{1}{1}$ and $v_2 = \binom{0}{1}$ determine whether $v_1\ and\ v_2$ span a vector in R^2 (4 marks)

Question FIVE (20 marks)

(a) Find the rank of the matrix $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{bmatrix}$ (4 marks)

(b) Solve by Cramer's rule the system of equations

$$x + 2y + 3z = 10,$$
$$2x - 3y + z = 1,$$
$$3x + y - 2z = 9,$$
(10 marks)

Q4ii Given $\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix}$ Find the value of λ (eigen values) such that $|\mathbf{A} - \lambda\mathbf{I}|=0$ (6 marks)